GRAPHICAL INTERPRETATION OF THE LOCAL HEAT TRANSFER CORRELATION EQUATION WITH COMBINED FREE AND FORCED CONVECTION

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An account is given, illustrated by an example, of a graphical interpretation and of an examination of the correlation equation for local heat transfer with combined free and forced convection in a tube in which hydrodynamic flow stabilization occurs but thermal stabilization is absent.

The literature presently contains papers in which correlations are made, to greater or less degree, of heat transfer data in conditions of free, mixed, and forced convection.

Watzinger and Johnson [2] have drawn attention to the fact that the cooling of a low-viscosity fluid in a laminar regime by the combined action of free and forced convection and at large values of the group Gr · Pr may be described in certain cases by the heat transfer equation for free convection.

By analytic solution of the system of differential equations for convective heat transfer, Ostroumov [3] obtained a solution which includes both free convection and forced flow as special cases.

Eckert, Drake, and Meteis [1, 6], in the coordinates Re—Gr · Pr d/l have generalized results of investigations for various regimes of forced, mixed, and free convection in tubes and channels and have established the regions of laminar, transitional, and turbulent regimes.

Sparrow, Eichhorn, and Gregg [4] have investigated analytically the conditions for simultaneous action of free and forced convection for heat transfer with non-established velocity fields, putting forward their final solution in the form of approximation relations recommended for use at characteristic ratios Gr/Re^2.

Shevchik [8] obtained a solution for simultaneous action of free and forced convection in laminar flow over a vertical flat plate by expansion in series of the flux and temperature functions in terms of the parameter Gr/Re^2.

From a generalization of the factors acting under conditions of both free convection and forced flow, Buznik and Vezlomtsev [5] proposed a generalized form of a correlation equation for heat transfer in external flow over bodies. Mayatskii, for an internal problem [7], attempted to represent the physical model of free convection in such a way as to reduce solution of the heat transfer problem to an application of the relations obtained for forced flow in tubes. The investigations of [5] and [7] have been reduced to computational relations.

Reference [10] described an investigation of local heat transfer in a tube for the case of combined action of free and forced laminar flow in conditions of prior hydrodynamic stabilization. The test data were processed in the form of the correlation relation

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1\right] = f\left(\mathrm{Pe}\frac{d}{l},\ \mathrm{Gr}\cdot\mathrm{Pr}\right),$$

which is shown in Fig. 1. The following characteristic special features of the experimental curve are evident from the figure:

1. The function $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$ is not monotonic. For a

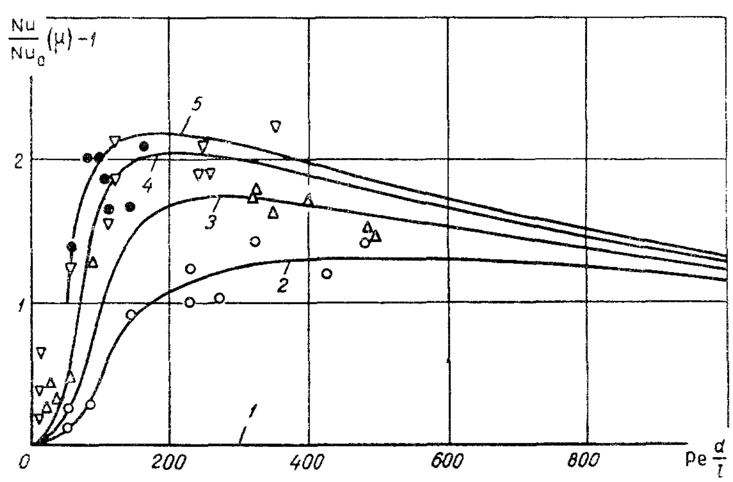

Fig. 1. Experimental curves for the function $\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right] = f\left(\mathrm{Pe}\frac{d}{l}\right)$ for $\mathrm{Gr}\cdot\mathrm{Pr} \leq 10^5$ (1), $2.8\cdot 10^5$ (2), $6.5\cdot 10^5$ (3), $1.5\cdot 10^6$ (4) and $3.4\cdot 10^6$ (5).

Table 1

Computation Sequence for the Quantity $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$

	$\mathrm{Pe}\frac{d}{l}$	10	30	50	100	300	500	1000	1500
	$\left(\mathrm{Pe}\frac{d}{l}\right)^{1.29}$	19.5	80	155	380	1500	3020	7410	12600
	$A = \frac{50.7\cdot 10^3}{\left(\mathrm{Pe}\frac{d}{l}\right)^{1.29}}$	2600	634	327	134	33.8	16.8	6.85	4.02
$\mathrm{Gr}\cdot\mathrm{Pr} = 5\cdot 10^5$,	$B = \frac{A}{(\mathrm{Gr}\cdot\mathrm{Pr})^{0.25}}$	98	23.9	12.3	5.13	1.27	0.63	0.26	0.15
$(\mathrm{Gr}\cdot\mathrm{Pr})^{0.25} = 26.5$,	$\sigma = \frac{D}{\mathrm{Pe}\frac{d}{l}}$	18.4	6.14	3.68	1.84	0.61	0.37	0.18	0.12
$[\mathrm{Gr}\cdot\mathrm{Pr} - 10^5]^{0.25} = 25.1$,	$\exp\sigma - 1$	10^8	467	38.8	5.31	0.85	0.44	0.20	0.13
$D = \frac{4620}{25.1} = 184.$	$\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1 = \frac{B}{\exp\sigma - 1}$	10^{-6}	0.05	0.32	0.97	1.48	1.44	1.26	1.14

Table 2

Computed Values of the Function $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$

Gr·Pr	$\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$ at $\mathrm{Pe}\frac{d}{l}$							
	10	30	50	100	300	500	1000	1500
$1.1\cdot 10^5$	0	0	0.002	0.07	0.51	0.61	0.64	0.615
$2\cdot 10^5$	0	0.005	0.086	0.51	1.16	1.17	1.08	1.00
$5\cdot 10^5$	0	0.051	0.318	0.967	1.48	1.44	1.26	1.14
10^6	0	0.134	0.54	1.22	1.65	1.52	1.36	1.21
10^7	0.01	0.78	1.38	1.87	1.91	1.67	1.40	1.25
10^8	0.26	1.73	2.17	2.27	2.00	1.75	1.42	1.25
10^9	1.17	2.58	2.69	2.54	2.11	1.76	1.43	1.26
10^{10}	2.48	3.20	3.05	2.74	2.14	1.77	1.45	1.27
10^{11}	3.62	3.53	3.23	2.81	2.22	1.82	1.47	1.28

constant value of Gr · Pr, the quantity $\frac{Nu}{Nu_0}(\mu)-1$ at first increases with increase of $Pe\frac{d}{l}$, reaches a maximum, and then begins to drop, tending to zero as $Pe\frac{d}{l}\to\infty$.

2. To the left of the maxima the family of curves with various Gr · Pr values forms a bundle converging to the origin of coordinates, as $Pe\frac{d}{l}$ decreases.

3. With increase of Gr · Pr the maximum is displaced towards the region of smaller values of $Pe\frac{d}{l}$.

4. With increase of Gr · Pr the quantity $\frac{Nu}{Nu_0}(\mu)-1$ clearly tends to some limit; the distance between neighboring curves decreases in going to a higher curve.

Analysis showed that the curves obtained cannot be described by a power-law correlation equation

$$Nu = c\cdot Re^m\cdot Pr^n\cdot Gr^k\cdot\left(\frac{d}{l}\right)^p\left(\frac{\mu_f}{\mu_w}\right)^r. \quad (1)$$

Reference [10] presents arguments on which an improved correlation equation is based. This equation has the following general form:

$$\frac{Nu}{Nu_0}(\mu)-1=\frac{c_1}{\left(Pe\frac{d}{l}\right)^{n_1}}\times\frac{1}{\left\{\exp\left\{c_2\Big/\left(Pe\frac{d}{l}\right)^{n_2}[Gr\cdot Pr-(Gr\cdot Pr)_{init}]^{n_3}\right\}-1\right\}}\cdot\frac{c_3}{(Gr\cdot Pr)^{n_4}}. \quad (2)$$

It was established, from the analysis of the equation performed and by calculation, that the following relations must hold:

$$n_1=n_0+n_2 \text{ and } n_3=n_4.$$

For the experimental conditions $n_0 = 0.29$, $n_2 = 1.0$, $n_1 = 1.29$, $n_3 = n_4 = 0.25$, $c_1 = 2380$, $c_2 = 4620$, $c_3 = 21.3$, Eq. (2) takes the following computational form:

$$\frac{Nu}{Nu_0}(\mu)-1=50.7\cdot10^3\Big/\left(Pe\frac{d}{l}\right)^{1.29}\times\left\{\exp\left[\frac{4620}{Pe\frac{d}{l}(Gr\cdot Pr-10^5)^{0.25}}\right]-1\right\}(Gr\cdot Pr)^{0.25}. \quad (2a)$$

The following limits for variation of the similarity criteria have been investigated experimentally: Re = 20–2500; Gr = $7\cdot10^3$–$1.2\cdot10^6$; Pr = 8.5–2.8; Gr · Pr = $5\cdot10^4$–$3.4\cdot10^6$.

Table 1 shows the computational sequence for the quantity $\frac{Nu}{Nu_0}(\mu)-1$ for the curve with Gr · Pr = $5\cdot10^5$.

However, if, in accordance with the ideas of reference [9] concerning the definite conservation of properties of the boundary layer, it is suggested that the laws described by Eq. (2) will hold even beyond the limit of the maximum values of Gr · Pr investigated experimentally, extrapolation of this equation is possible.

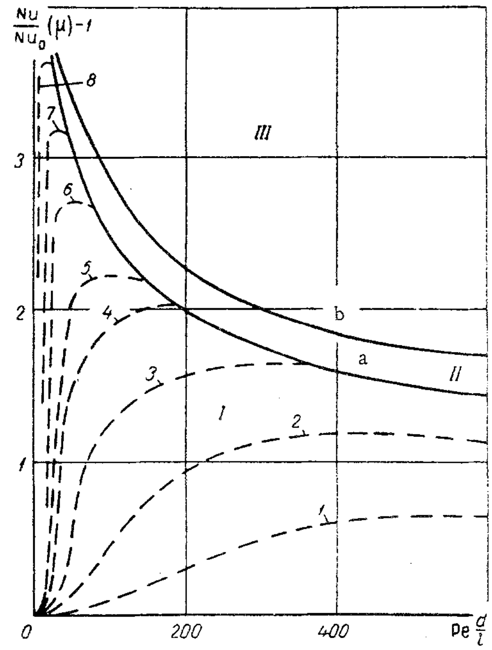

Fig. 2. Calculated curved constructed from Eq. (2a) with Gr · Pr = $1.1\cdot10^5$ (1), $2\cdot10^5$ (2), 10^6 (3), 10^7 (4), 10^8 (5), 10^9 (6), 10^{10} (7), and 10^{11} (8); the first boundary curve—from Eq. (7a)—a, the second boundary curve—from equation (10a)—b (regimes: I—laminar, II—transitional, III—turbulent).

Table 2 shows calculations of $\frac{Nu}{Nu_0}(\mu)-1$ for curves with values of Gr · Pr from $1.1\cdot10^5$ to 10^{11}.

A correlation equation of type (2) allows us not only to determine the heat transfer rate in the laminar flow region for various relative values of $Pe\frac{d}{l}$ and Gr · Pr, but also to define the regions of laminar, transition, and turbulent flow over the whole range of possible active conditions of free convection and forced flow. We shall designate

$$\frac{c_2}{\left(Pe\frac{d}{l}\right)^{n_2}[Gr\cdot Pr-(Gr\cdot Pr)_{init}]^{n_3}}=\sigma. \quad (3)$$

As is known, the quantity $\exp\sigma - 1$ may be expanded in a Taylor series, i.e.,

$$\exp\sigma-1=\frac{\sigma}{1!}+\frac{\sigma^2}{2!}+\frac{\sigma^3}{3!}+\ldots.$$

For small values of σ we may restrict attention to only the first term of the right side. Hence it follows that

$$\lim\left\{\exp\left\{\frac{c_2}{\left(Pe\frac{d}{l}\right)^{n_2}[Gr\cdot Pr-(Gr\cdot Pr)_{init}]^{n_3}}\right\}-1\right\}=$$

$$= \frac{c_2}{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_2}[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]^{n_3}},$$

$$\text{as} \quad \left\{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_2}[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]^{n_3}\right\} \to \infty. \qquad (4)$$

To determine the equation of the boundary curve between the laminar and transition regimes, as a reference we made use of a point which has frequently been defined by investigators. It was assumed that the laminar regime with Re = 2150, Pr = 3.8, Pe d/l = = 525, Gr · Pr = $1.5 \cdot 10^6$ will undergo transition. The ratio $(l/d)_{\mathrm{meas}}$ was equal to 15.6 [10] in the conditions of the test.

Table 3

Coordinates of the Boundary Curves

Gr·Pr	$(\mathrm{Gr}\cdot\mathrm{Pr}-10^5)^{0.25}$	$\left[\mathrm{Pe}\frac{d}{l}\right]_1$	$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_1$	$\left[\mathrm{Pe}\frac{d}{l}\right]_2$
$1{,}1\cdot10^5$	10	1805	1.07	8370
$2\cdot10^5$	17.8	1030	1.26	4700
$5\cdot10^5$	25.1	720	1.40	3340
10^6	30.9	584	1.48	2710
10^7	56.2	321	1.76	1490
10^8	100	180	2.09	837
10^9	178	101	2.47	470
10^{10}	316	57.2	2.91	265
10^{11}	562	32.1	3.44	149

As is usual we shall call the curve between the laminar and transitional regimes the first boundary curve. For this curve

$$\sigma_1 = \frac{4620}{2150\cdot3.8\dfrac{1}{15.6}[(15-1)\cdot10^5]^{0.25}} = 0.256. \qquad (3a)$$

Then $\exp\sigma_1 - 1 = 0.30$, $\exp\sigma_1 - 1)/\sigma_1 = 0.30/0.256 = 1.17$.

Therefore, for the limiting form of the equation we may write

$$\exp\sigma_1 - 1 = 1.17\sigma_1. \qquad (5)$$

Substitution of the quantity $1.17\sigma_1$ in place of $\exp\sigma_1 - 1$ brings Eq. (2) into the form

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_1 = \frac{c_1}{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_1}}\cdot$$

$$\cdot\frac{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_2}[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]^{n_3}}{1.17c_2}\cdot\frac{c_3}{(\mathrm{Gr}\cdot\mathrm{Pr})^{n_4}}. \qquad (6)$$

If we neglect the quantity $(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{initial}}$ (for Gr · Pr = $1.5\cdot10^6$ and $(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{initial}} = 10^5$ the error in the computations was about 1%), we may reduce the quantities $[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]^{n_3}$ and $(\mathrm{Gr}\cdot\mathrm{Pr})^{n_4}$, since $n_3 = n_4$. Then we obtain

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_1 = \frac{c_1c_3}{1.17c_2}\cdot\frac{1}{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_1-n_2}}. \qquad (7)$$

Equation (7) is the equation of the boundary curve between the laminar and transitional regimes throughout the range of Reynolds numbers characteristic for laminar isothermal flow. It has the form of a generalized hyperbola.

For the boundary curve we may calculate the relative values of Pe d/l and Gr · Pr. Since the value of σ_1 is constant in this case, by using Eq. (3), we may write

$$\left[\mathrm{Pe}\frac{d}{l}\right]_1 = \left\{\frac{c_2}{\sigma_1[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]_1^{n_3}}\right\}^{1/n_2}, \qquad (8)$$

or

$$[\mathrm{Gr}\cdot\mathrm{Pr}]_1 = \left[\frac{c_2}{\sigma_1\left(\mathrm{Pe}\frac{d}{l}\right)_1^{n_2}}\right]^{1/n_3} + (\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}. \qquad (9)$$

The numerical form of Eqs. (7–9) for the conditions of the experiment conducted will be as follows:

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_1 = \frac{2380\cdot21.3}{1.17\cdot4620}. \qquad (7a)$$

$$\frac{1}{\left(\mathrm{Pe}\frac{d}{l}\right)^{(1.29-1)}} = \frac{9.4}{\left(\mathrm{Pe}\frac{d}{l}\right)^{0.29}}, \qquad (8a)$$

$$\left[\mathrm{Pe}\frac{d}{l}\right]_1 = \frac{18\,000}{(\mathrm{Gr}\cdot\mathrm{Pr}-10^5)^{0.25}},$$

$$[\mathrm{Gr}\cdot\mathrm{Pr}]_1 = \left(18\,000\Big/\mathrm{Pe}\frac{d}{l}\right)^4 + 10^5. \qquad (9a)$$

Similarly, we may find the equation of the boundary curve between the transitional and turbulent regimes, which we shall conventionally call the second boundary curve. Taking the values Re = 10 000, Pr = 3.8, Pe d/l = = 2440, Gr · Pr = $1.5\cdot10^6$, we shall have for these conditions

$$\sigma_2 = \frac{4620}{2440\cdot\dfrac{1}{15.6}[(15-1)\cdot10^5]^{0.25}} = 0.0552,$$

$$\exp\sigma_2 - 1 = 0.0566.$$

Thus,

$$\exp\sigma_2 - 1 = 1.025\sigma_2,$$

and the equation of the second boundary curve takes the form

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_2 = \frac{c_1c_3}{1.025c_2}\cdot\frac{1}{\left(\mathrm{Pe}\frac{d}{l}\right)^{n_1-n_2}}. \qquad (10)$$

For the conditions under which the experiments were conducted

$$\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu)-1\right]_2 = \frac{10.7}{\left(\mathrm{Pe}\frac{d}{l}\right)^{0.29}}. \qquad (10a)$$

In an analogous way equations (8) and (9) for the second boundary curve may be written as

$$\left[\mathrm{Pe}\frac{d}{l}\right]_2 = \left\{\frac{c_2}{\sigma_2[\mathrm{Gr}\cdot\mathrm{Pr}-(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}]_2^{n_3}}\right\}^{1/n_2}, \qquad (11)$$

$$[\mathrm{Gr}\cdot\mathrm{Pr}]_2 = \left[\frac{c_2}{\sigma_2\left(\mathrm{Pe}\frac{d}{l}\right)_2^{n_2}}\right]^{1/n_3} + (\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}. \qquad (12)$$

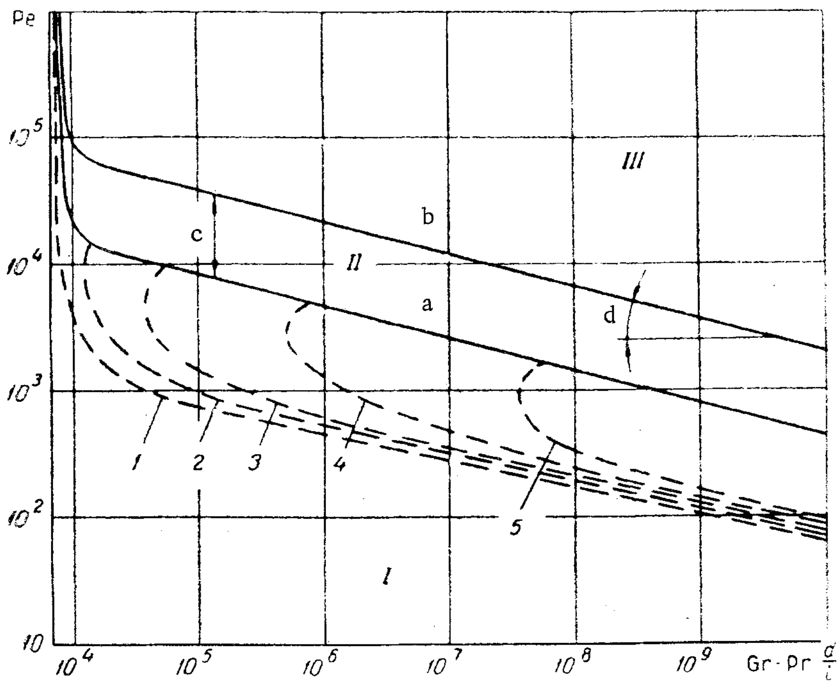

Fig. 3. Regimes of local convective heat transfer with simultaneous action of free and forced convection (l/d = 15.6): 1, 2, 3, 4, and 5—curves of constant heat transfer intensity for $\frac{Nu}{Nu_0}$ (x) respectively 1.5, 2, 2.5, 3, and 3.5; a—the first boundary curve, constructed from equation (14a); b—the second boundary curve—from equation (14b). Regimes: I—laminar, II—transitional, III—turbulent.

For our case

$$\left[\mathrm{Pe}\frac{d}{l}\right]_2 = \frac{84000}{(\mathrm{Gr}\cdot\mathrm{Pr} - 10^5)^{0.25}}, \tag{11a}$$

$$[\mathrm{Gr}\cdot\mathrm{Pr}]_2 = \left(\frac{84000}{\mathrm{Pe}\frac{d}{l}}\right)^4 + 10^5. \tag{12a}$$

It should be noted that equation (10), while it describes analytically the boundary curve between the transitional and turbulent regime, cannot however tell us to which values of Cr · Pr the values of $\left[\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1\right]_2$ obtained correspond. Since the turbulent regime is characterized by the presence of fluctuations, the fluctuating component leads to a different dependence of heat transfer on the Reynolds number than in the laminar regime. Naturally, the equation derived from experimental data for the laminar regime will understate the heat transfer intensity in the turbulent regime.

Table 3 gives calculated relative values of Gr · Pr, $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$ and Pe d/l for the first and second boundary curves. A graph of the equations $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1 = f\left(\mathrm{Pe}\frac{d}{l}, \mathrm{Gr}\cdot\mathrm{Pr}\right)$, drawn in linear coordinates from the data of Tables 2 and 3, is shown in Fig. 2. The x axis on the graph corresponds to forced laminar flow (Gr · Pr $\leq 10^5$), and the y axis coincides with the free convection regime $\left(\mathrm{Pe}\frac{d}{l} = 0\right)$. The graph shows calculated curves of the relation $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$, as well as the first and second boundary curves. Since the laminar regime has been investigated experimentally, the curves of $\frac{\mathrm{Nu}}{\mathrm{Nu}_0}(\mu) - 1$ are given only below the first boundary curve.

The graph shows that with decrease of $\mathrm{Pe}\frac{d}{l}$ the transition regime region decreases, i.e., the laminar and turbulent regimes appear to draw closer together. This must evidently be the reason for the absence of the transitional regime in free convection, it having proved impossible to observe this with the inherent accuracy of the experimental investigations.

It is interesting to note that in local heat transfer the phenomenon of free convection corresponds to the case when Gr · Pr = ∞. However, in the actual conditions the produce Gr · Pr is finite. The cause of this phenomenon is evidently that in free convection there is always superimposed some degree of forced flow.

For a qualitative comparison of the computed equation with the generalized graph presented in the monograph of Eckert and Drake [1], the parametric equation [2] has been recalculated in the form of the relation

$$\mathrm{Pe} = f\left(\mathrm{Gr}\cdot\mathrm{Pr}\cdot\frac{d}{l}\right), \tag{13}$$

which is is more convenient to construct in logarithmic coordinates. Transforming and taking logarithms of Eq. (3) we obtain

$$\lg \mathrm{Pe} = \frac{1}{n_2}\lg\left[\frac{c_2}{\sigma}\left(\frac{d}{l}\right)^{(n_3-n_2)}\right] - \frac{n_3}{n_2}\lg\left[\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l} - \left(\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}\right)_{\mathrm{init}}\right]. \tag{14}$$

The first term on the right side here is a constant coefficient. The negative sign ahead of the second term on the right indicates that the quantity Pe decreases with increase of $\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}$. While $\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}$ tends to $\left(\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}\right)_{\mathrm{init}}$, the first part of Eq. (14) tends to infinity.

When the quantity $\left(\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}\right)_{\mathrm{init}}$ may be neglected in comparison with $\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}$, (14) is an equation of a straight line in logarithmic coordinates. Under these conditions the coefficient n_3/n_2 is the tangent of the angle of slope of the line with respect to the x axis.

In determining the first boundary curve we must substitute the numerical value σ_1 in place of σ, in determining the second boundary curve the numerical value σ_2 is substituted. In our case Eq. (14) acquires the form:

for the first boundary curve

$$\lg \mathrm{Pe}_1 = 5.152 - 0.25\lg\left[\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l} - 6.4\cdot 10^3\right], \tag{14a}$$

for the second boundary curve

$$\lg \mathrm{Pe}_2 = 5.819 - 0.25\lg\left[\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l} - 6.4\cdot 10^3\right]. \tag{14b}$$

Comparison of Eqs. (14a) and (14b) shows that the first and second boundary curves are equidistant in logarithmic coordinates. Analytically this means that

$$|\lg \mathrm{Pe}_2 - \lg \mathrm{Pe}_1|_{\mathrm{Gr}\cdot\mathrm{Pr}} = \frac{1}{n_2}\lg\frac{\sigma_1}{\sigma_2} = \mathrm{const}. \tag{15}$$

In our case

$$|\lg \mathrm{Pe}_2 - \lg \mathrm{Pe}_1|_{\mathrm{Gr}\cdot\mathrm{Pr}} = \lg\frac{0.256}{0.0552} = 0.667. \tag{15a}$$

It was established by calculation that the quantity $\left(\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}\right)_{\mathrm{init}} = 6.4\cdot 10^3$ may be neglected, when (Gr · Pr$(d/l)) \geq 3.2\cdot 10^4$. The deviation from a straight line relation in the case when $\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l} \to \left(\mathrm{Gr}\cdot\mathrm{Pr}\frac{d}{l}\right)_{\mathrm{init}}$, is due to peculiarities in the structure of the barometric equation, which is constituted in such a way that the group Gr·Pr is counted from the value $(\mathrm{Gr}\cdot\mathrm{Pr})_{\mathrm{init}}$, at which the influence of free convection on heat transfer commences [10].

The boundary curves and the characteristic heat transfer regions are shown in Fig. 3. Curves of constant heat transfer intensity are shown in the laminar region, expressed in terms of the quantity $\frac{Nu}{Nu_0}(\mu)$, these being obtained from Fig. 2.

Comparison of Fig. 3 with the generalized graph of Eckert and Drake leads to the conclusion that they are qualitatively in agreement. This means that the barometric equation (2) allows us to describe analytically the whole field of possible regimes of convective heat transfer, this being the difference in principle between (2) and equations of the type (1), which describe heat transfer by means of a power-law function of similarity criteria.

NOTATION

Nu_0—local value of Nusselt number in forced laminar flow; Nu—local value of Nusselt number in superposition of free convection on forced laminar flow; Gr—Grashof number, referred to the mean fluid temperature: Re—Reynolds number; Pr—Prandtl number: 1/d—relative distance from the origin to the measurement section: $(\mu) = (\mu_f/\mu_w)_0^{0.14}/(\mu_f/\mu_w)^{0.14}$ —correction taking into account the direction of the heat flux; $c_1, c_2, c_3, n_1, n_2, n_3, n_4$—constant coefficients, chosen according to the experimental data; n_0—exponent of $Pe\frac{d}{l}$ in the barometric heat transfer equation for forced laminar flow $\left(Nu_0 = c_0\left(Pe\frac{d}{l}\right)^{n_0}\left(\frac{\mu_f}{\mu_w}\right)_0^{0.14}\right)$.

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